More Is Different... But How Many Is Different?

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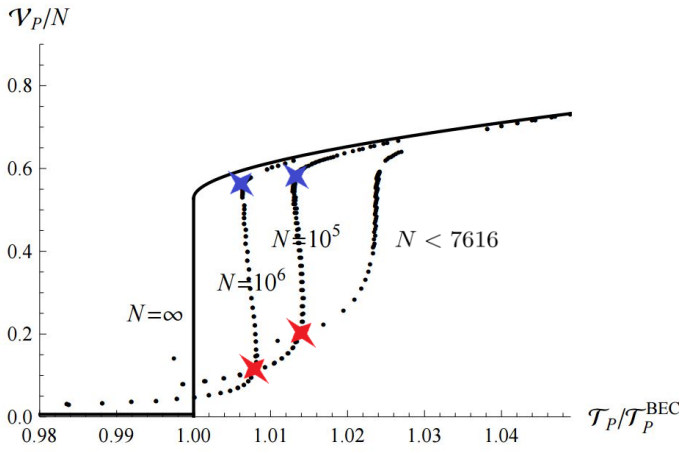

⋆ **Supercooling Point**

$$T_P^\star/T_P^{\rm BEC} \simeq 1+\tfrac{\pi^3}{60}\left[(T_P^{\rm BEC})^5/T_V^{\rm BEC}\right]^{\frac{1}{2}} N^{-\frac{1}{3}}$$

$$V_P^\star \simeq (T_V^{\rm BEC}/T_P^{\rm BEC})^{\frac{3}{2}}\left(N+\tfrac{\pi}{4}T_V^{\rm BEC}\,N^{\frac{2}{3}}\ln N\right)$$

⋆ **Superheating Point**

$$T_P^\star/T_P^{\rm BEC} \simeq 1+\tfrac{1}{150}\left(\tfrac{\pi^{15}}{15}\right)^{\frac{1}{4}}(T_P^{\rm BEC})^{\frac{5}{2}}\,N^{-\frac{1}{4}}$$

$$V_P^\star \simeq 8\left(\tfrac{15}{\pi^3}\right)^{\frac{3}{4}}(T_P^{\rm BEC})^{-\frac{3}{2}}\,N^{\frac{3}{4}}$$

where (see [3] for details)

$$T_P = k_{\rm B}TP^{-\frac{2}{5}}\left(\tfrac{2m}{\pi^2\hbar^2}\right)^{\frac{3}{5}} \qquad V_P = V\left(P\tfrac{2m}{\pi^2\hbar^2}\right)^{\frac{3}{5}}$$

The isobar of the standard ideal Bose gas, confined in a cubic box, zigzags if the number of particles is no less than 7616 (canonical ensemble, exact result) [1]. This critical number can be viewed as the *quantum characteristic of cube* or Jacobi theta function. Depending on the shape and dimension of the container, yet not size, the critical number varies but always exists [4, 5]. Analytic methods for large N reveal the finite effect of Avogadro number [3], and predict isobaric critical exponents even for interacting real molecules [2], which are in good agreement with experiments [6, 7]. Liquid-gas phase transitions may occur essentially due to the identical nature of particles. In the sense of Thomas Kuhn, these works might be viewed as "anomaly" within the current paradigm of taking the thermodynamic limit for realizing discrete phase transitions. A finite system cannot feature any discrete phase transition if the density is fixed, but it can do so well if the alternative 'isobaric' constraint is imposed: without changing the density it is hard to boil water but it becomes easy under constant pressure, such as 100°C at 1 atm.

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[6] W. Cho, D. H. Kim and JHP, *"Isobaric Critical Exponents: Test of Analyticity against NIST Reference Data,"* Front. Phys. **6** (2018) 112 arXiv:1612.06532 [cond-mat.stat-mech].

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